AIAA JOURNAL
Vol. 30, No. 11, November 1992

# Unsteady Laminar Compressible Swirling Flow with Massive Blowing

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## Abstract

THE study of swirling boundary layers is of considerable importance in many rotodynamic machines such as rockets, jet engines, swirl generators, swirl atomizers, arc heaters, etc. For example, the introduction of swirl in a flow acceleration device such as a nozzle in a rocket engine promises efficient mass flow control. In nuclear rockets, swirl is used to retain the uranium atoms in the rocket chamber. With these applications in mind, Back[1] and Muthanna and Nath[2] have obtained the similarity solutions for a low-speed three-dimensional steady laminar compressible boundary layer with swirl inside an axisymmetric surface of variable cross section.

The aim of the present analysis is to study the effect of massive blowing rates on the unsteady laminar swirling compressible boundary-layer flow of an axisymmetric body of arbitrary cross section when the freestream velocity and blowing rate vary with time. The type of swirl considered here is that of a free vortex superimposed on the longitudinal flow of a compressible fluid with variable properties. The analysis is applicable to external flow over a body as well as internal flow along a surface. For the case of external flow, strong blowing can have significant use in cooling the surface of hypervelocity vehicles, particularly when ablation occurs under large aerodynamic or radiative heating, but there may not be such an important application of strong blowing in the case of internal flow. The governing partial differential equations have been solved numerically using an implicit finite difference scheme with a quasilinearization technique.[3] High temperature gas effects, such as radiation, dissociation, and ionization, etc., are not investigated. The nomenclature is usually that of Ref. 4 and is listed in the full paper.

## Contents

The unsteadiness in the flowfield is introduced by the freestream velocity distribution that varies arbitrarily with time. We have considered the Prandtl number as constant because in most problems of compressible fluid its variations in the boundary layer are small. We have also assumed that the external flow is homentropic and the surface is maintained at a constant temperature. Under the foregoing assumptions, the unsteady boundary-layer equations can be expressed in nondimensional form as[1,2,4]

$$
\begin{aligned}
&(NF')' + \phi fF' + \beta\phi[(\rho_e/\rho) - F^2] + \alpha\phi[(\rho_e/\rho) - S^2] \\
&\quad + \beta\{\phi^{-1}\phi_t^*[(\rho_e/\rho) - F] - F_t^*\} \\
&\quad - 2t^*\{\phi_t^*[\rho_e/\rho - F^2 + fF'] + \phi(f_t^*F' - FF_t^*)\} = 0 \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&(NS')' + \phi fS' + \beta\{\phi^{-1}\phi_t^*[(\rho_e/\rho) - S] - S_t^*\} \\
&\quad - 2t^*\{\phi_t^*[(\rho_e/\rho) - SF + fS'] + \phi(f_t^*S' - FS_t^*)\} = 0 \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&(NG')' + Pr\phi fG' - 2(1 - Pr)E\{N[FF' + (v_s/u_s)^2 SS']\}' \\
&\quad - 2Prt^*[\phi_t^* fG' + \phi(f_t^*G' - FG_t^*)] - \beta Pr\, G_t^* = 0 \qquad (3)
\end{aligned}
$$

We have used the following transformations:

$$\eta = (2\xi)^{-1/2}\rho_e u_s r\int_0^z (\rho/\rho_e)\,dz$$

$$\xi = \int_0^x \rho_e\mu_e u_s r^2\,dx, \qquad t^* = (du_s/dx)t$$

$$\psi(x, z, t) = (2\xi)^{1/2} f(\eta, t^*)\phi(t^*)$$

$$u_e = u_s\phi(t^*), \qquad v_e = v_s\phi(t^*)$$

$$\rho u = r^{-1}(\partial\psi/\partial z)$$

$$\rho w = -r^{-1}[(\partial\psi/\partial x) + \int_0^z r(\partial\rho/\partial t)\,dz]$$

$$u = u_e f' = u_e F$$

$$v = v_e S, \quad H = H_e G, \quad H = h + (u^2 + v^2)/2$$

$$E = (u_e^2/2H_e) = (u_s^2/2H_e)\phi^2 = m\phi^2$$

$$\beta = (2\xi/u_s)(du_s/d\xi)$$

$$\alpha = -(2\xi/r)(dr/d\xi)(v_s/u_s)^2 = \beta(v_s/u_s)^2$$

$$(\;)' = \partial/\partial\eta$$

$$\rho\alpha h^{-1}, \quad \mu\alpha h^{\omega}, \quad N = \rho\mu/(\rho_e\mu_e) = (h/h_e)^{\omega-1}$$

$$\frac{\rho_e}{\rho} = \frac{h}{h_e} = \frac{H - (u^2/2) - (v^2/2)}{H_e - (u_e^2/2) - (v_e^2/2)}$$

$$= \frac{G - (u_e^2/2H_e)\,F^2 - (v_e^2/2H_e)\,S^2}{1 - (u_e^2/2H_e) - (v_e^2/2H_e)}$$

The boundary conditions imposed on the set of equations (1-3) at any particular time $t^* \geq 0$ are

$$
\left.
\begin{aligned}
&F = S = 0, \quad G = G_w \quad \text{at } \eta = 0 \\
&F \to 1, \quad S \to 1, \quad G \to 1 \quad \text{as } \eta \to \infty
\end{aligned}
\right\} \qquad (4)
$$

where

$$f = \int_0^\eta F\,d\eta + f_w$$

$f_w$ is given by

$$\phi f_w - 2t^*[(f_t^*)_w\phi + \phi_t^* f_w] = A$$

Received Aug. 5, 1991; synoptic received Nov. 8, 1991; accepted for publication Nov. 13, 1991. Full paper available from AIAA Library, 555 W. 57th Street, New York, NY 10019. 

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and

$$A = (\rho w)_w (2\xi)^{1/2}/(\rho_e \mu_e u_s r)$$

Here we have taken $u_s = b/r$, $r \alpha \xi^{-\beta/2}$, $(\rho w)_w \alpha \xi^{-1/2}$, and the free-vortex relation[1] $\Gamma = r v_s$ ($\Gamma$ is circulation) for the semisimilar solution so that $\alpha$, $\beta$, and $A$ are constants and $A > 0$ or $A < 0$ according to whether there is suction or injection. In particular for low-speed flow, the surface radius must vary according to $r = c\ \xi^{-\beta/2}$ with $c =$ const and the freestream velocity variation is given by $u_s = a\ \xi^{\beta/2}$, with $a =$ const.[1] For $\beta > 0$, the radius decreases with increasing $\xi$ or $x$ along the surface, corresponding to a converging channel. We have also considered the product of density viscosity at the edge of the boundary layer $\rho_e \mu_e$ as constant. It may be remarked that the potential flow solution represented by $u_e$ and $v_e$ holds good under the assumption that $\rho_e$ is a constant that is valid for low-speed flows. Also, as a first approximation, it can be used for high-speed flows without introducing any appreciable error.[4] The initial conditions are given by the steady-state equations that can be obtained from Eqs. (1–3) by putting $t^* = 0$, $\phi = 1$, $\phi_t^* = F_t^* = S_t^* = G_t^* = 0$.

By introduction of a length $\bar{\xi}$ defined by

$$\bar{\xi} = \left( \int_0^x \rho_e \mu_e u_s r^2 \, dx \right) \Bigg/ \left( \rho_e \mu_e u_s r^2 \right)$$

the longitudinal skin friction $C_f$, tangential skin friction $\bar{C}_f$, and Stanton number $St$ can be written as[1,2]

$$C_f = (2\tau_x/\rho_e u_s^2) = (Re_{\bar{\xi}})^{-1/2} \bar{F}_w', \quad \bar{C}_f = (2\tau_y/\rho_e u_s^2) = (Re_{\bar{\xi}})^{-1/2} \bar{S}_w'$$

$$St = q_w/[\rho_e u_s (H_e - H_w)] = (Re_{\bar{\xi}})^{-1/2} \bar{G}_w'$$

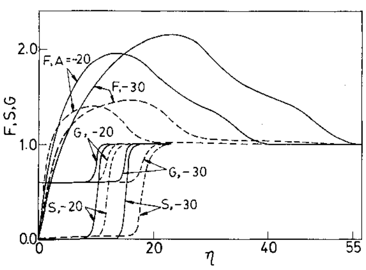


**Fig. 1 Effect of injection on velocity and enthalpy profiles ($F,S,G$) for $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon = 0.05$, $G_w = 0.6$, $\omega = 1.0$, $\beta = 5$, $(u_s^2/2H_e) = 0.02$, and $(v_s/u_s)^2 = 2$; ———, $t^* = 0$; - - -, $t^* = 1.0$.**

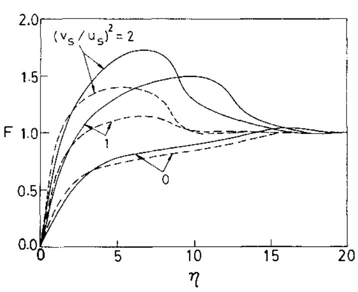


**Fig. 2 Effect of swirl parameter on longitudinal velocity profiles ($F$) for $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon = 0.20$, $G_w = 0.6$, $\omega = 0.50$, $\beta = 5$, $(u_s^2/2H_e) = 0.02$, and $A = -10$; ———, $t^* = 0$; - - -, $t^* = 1.0$.**

where

$$\bar{F}_w' = 2^{1/2} \phi\ N_w f_w'', \quad \bar{S}_w' = 2^{1/2} (v_s/u_s)\ \phi\ N_w\ S_w'$$

$$\bar{G}_w' = 2^{-1/2}\ Pr^{-1}\ (1 - G_w)^{-1}\ N_w G_w'$$

and $Re_{\bar{\xi}}$ is the Reynolds number $(\rho_e u_s \bar{\xi}/\mu_e)$.

The set of equations (1–3) under boundary conditions (4) and initial conditions given by the steady-state equations has been solved numerically with the help of an implicit finite difference scheme in combination with the quasilinearization technique.[3] To fix the step sizes for computation, they have been optimized using Richardson's extrapolation formula. Computations have been carried out for various values of the parameters and for two different unsteady freestream velocity distributions given by

$$\phi(t^*) = 1 \pm \epsilon\ t^{*2}, \quad (\epsilon > 0), \quad \phi(t^*) = [1 + \epsilon_1 \cos(\omega^* t^*)]/(1 + \epsilon_1)$$

but the results are presented here only for $\phi(t) = 1 + \epsilon t^{*2}$. The full paper contains the detailed results for all of the cases. We have compared our skin friction and heat transfer results with those of Refs. 1,2, and 5 and with those of some other related works and found them in excellent agreement. The maximum difference is less than 1.2%. The comparison is not shown here to save the space (given in the full paper).

The effects of swirl parameter $[(v_s/u_s)^2]$ and injection parameter ($A < 0$) on the longitudinal velocity profile $F$ for $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$ are displayed in Figs. 1 and 2. Figure 1 also shows the effect of injection parameter on swirl and enthalpy profiles $S$ and $G$. It is observed that there is an overshoot in the longitudinal velocity profile $F$ in the presence of swirl as well as for large injection. Swirl and injection increase the magnitude of the velocity $F$ overshoot, but it decreases with the increase of time $t^*$. The overshoot is not present in swirl velocity and enthalpy profiles $S$ and $G$. Also, it is clear from Fig. 1 that, for large blowing rates, there is a thick inner layer close to the surface where the changes in the values of swirl velocity and enthalpy $S$ and $G$ are very small and a relatively thin outer viscous layer where the transition from the inner layer to the inviscid external stream takes place rapidly. To show the phenomenon more clearly, we also present some of the results quantitatively. For example, for $A = -30$ at the initial time $t^* = 0$, the velocity inside the boundary layer attains approximately 2.15 times its freestream value (i.e. 115% overshoot) and at time $t^* = 1.0$, the overshoot becomes 46%. Also, for $t^* = 0$ and $A = -10$, the overshoot in $F$ increases from 4 to 74% as $(v_s/u_s)^2$ increases from 0 to 2.

The effects of $\omega$, $A$, $(v_s/u_s)^2$, and $\beta$ on the skin friction and heat transfer $(\bar{F}_w', \bar{S}_w', \bar{G}_w')$ have also been studied, but not shown here for the sake of brevity. Skin friction and heat transfer increase with $\beta$ and $(v_s/u_s)^2$ but decrease with the increase of injection and $\omega$. This is true for all time $t^* \geq 0$. Also, the effect of $\omega$ becomes less pronounced as the blowing rate increases, and for large blowing, the effect is almost negligible. On the other hand, the effect of swirl parameter is found to be more pronounced on the skin friction $(\bar{F}_w', \bar{S}_w')$ than on the heat transfer $(\bar{G}_w')$.

## References


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